$$x = 2pq = 8mn(m^2 + n^2), \qquad y = p^2 - q^2 = (4mn)^2 - (m^2 + n^2)^2,$$

$$z = \frac{pq(r^2 - s^2)}{rs} = \frac{2mn[4(m^2 - n^2)^2 - (m^2 + n^2)^2]}{m^2 - n^2}.$$

Reducing the foregoing values of x, y, z to a common deominator $m^2 - n^2$ and then discarding it, we finally have

$$x = 8mn(m^2 - n^2)(m^2 + n^2),$$
$$y = (m^2 - n^2)[(4mn)^2 - (m^2 + n^2)^2],$$
$$z = 2mn[4(m^2 - n^2)^2 - (m^2 + n^2)^2].$$

Take $m = 2$, $n = 1$ and we get $x = 240$, $y = 117$, $z = 44$, the smallest values known. The diagonals of the faces are 267, 244, 125.

Also solved by C. B. HALDEMAN and L. E. LUNN.

2664. Proposed by J. W. NICHOLSON, Baton Rouge, La.

Find the sum of the series, $\frac{1}{3} - \frac{2}{15} + \frac{3}{35} - \cdots + (-1)^{n+1} \dfrac{n}{(2n-1)(2n+1)}$.

SOLUTION BY S. W. REAVES, University of Oklahoma.

The exponent of -1 in the general term should obviously be $n + 1$. Making this correction, the general term may be written,

$$(-1)^{n+1} \frac{n}{(2n-1)(2n+1)} = \tfrac{1}{2}(-1)^{n+1} \left(\frac{n}{2n-1} - \frac{n}{2n+1} \right).$$

Let S denote the required sum. Then

$$S = \frac{1}{2}\left[\sum_{i=1}^{n} (-1)^{i+1} \frac{i}{2i-1} + \sum_{i=1}^{n} (-1)^{i+2} \frac{i}{2i+1} \right]$$

$$= \frac{1}{2}\left[\left(1 - \tfrac{2}{3} + \tfrac{3}{5} - \cdots + (-1)^{n+1} \frac{n}{2n-1}\right) + \left(\tfrac{1}{3} + \tfrac{2}{5} - \cdots + (-1)^{n+2} \frac{n}{2n+1}\right) \right]$$

$$= \frac{1}{2}\left[1 - 1 + 1 - 1 + \cdots + (-1)^{n+2} \frac{n}{2n+1} \right].$$

If n be even, $S = \dfrac{n}{4n+2} = \frac{1}{4}$ when n is infinite.

If n be odd, $S = \dfrac{n+1}{4n+2} = \frac{1}{4}$ when n is infinite.

Also solved by BANCROFT H. BROWN, E. H. WORTHINGTON, E. B. ESCOTT, GEORGE F. WILDER, PAUL CAPRON, HORACE OLSON, ELIJAH SWIFT, and ARNOLD DRESDEN.

2665. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

A telegraph wire, which weighs 1/10 lb. per yard, is stretched between poles on a level ground so that the greatest dip of the wire is 3 feet. Find, approximately, the distance between the poles when the tension at the lowest point of the wire is 140 pounds.

SOLUTION BY G. PAASWELL, New York City.

MacMahon's *Hyperbolic Functions*, pages 47 to 51 discusses the two cases here, (*a*) when the wire is taken as inextensible, and (*b*) as extensible. Under (*a*) $y/c = \sec \varphi$; $x/c = gd^{-1} \varphi = \log_e \tan (\pi/4 + \varphi/2)$. The origin is the point midway between the poles and a distance c below the lowest point of the wire. Use the yard as the unit of length. $c = H/w = 140/0.1 = 1400$. $y = 1 + c = 1401$. Whence, $\varphi = 2° \; 9' \; 54''$. Thence, $x = 52.913$ and since $L = 2x$, the re-

quired length is 105.826 yards. Note here that if the weight is taken as distributed along the horizontal, i. e., along the x-axis instead of along the wire there is obtained by simple moments $c\,dy = x\,dx$ and with the origin as before except that the low point is taken instead of a point c below, there is obtained the parabolic relation $x = \sqrt{2cy}$, with $y = 1$. Hence, $x = \sqrt{2800} = 52.915$ and $L = 105.83$ yds., which is sufficiently close for this problem. As a matter of interest, it is seen that since $y/c = \cosh x/c$, the origin a distance c below the low point, by expanding the cosh to two terms, $1 + x^2/2c^2$, and replacing y by $y - c$ the parabolic relation above is obtained, which is a measure of the approximation of this method of solving the catenary.

(*b*) Here $y/c = \sec\varphi + \frac{1}{2}m\tan^2\varphi$; $x/c = gd^{-1}\varphi + m\tan\varphi$. $m = 1H$, where 1 is the unit extension of the wire. Since the wire weighs 0.1 pound per yd., taking steel as weighing 10.2 pounds per sq. in. per yd., and the modulus of elasticity as 30,000,000 pounds per sq. in., we have since $1 = H/AE$, where a is the area of the section of the wire and E the modulus, $m = 6664 \times 10^{-5}$. Expressing the y equation as a quadratic in $\sec\varphi$ and solving for φ, $\varphi = 2°\ 5'\ 54''$. Substituting this value in the equation for x and again replacing the gudermanian by its logarithmic equivalent we get $x = 54.74$ and $L = 109.58$ yds. Do not mistake the difference between this value and the above as the extension of the wire. Its meaning is that when allowance is made for extension it is necessary to spread the supports above the difference apart in order to maintain the deflection of one yard. The parabolic expression for this case is more complicated than the exact.

Also solved by J. V. Balch, Horace Olson, A. R. Nauer, Paul Capron, and Louis G. Pooler.

2666. Proposed by W. WOOLSEY JOHNSON, Annapolis, Md.

Ten equations between five quantities, x_1, x_2, x_3, x_4, x_5 being written as follows: $x_1 = 1 - x_3x_4$ and four others formed by the cyclic interchange of the suffixes; also $x_5x_1x_2 = x_5 + x_2 - 1$ and four others formed by the cyclic interchange; prove that only three of these equations are independent. In other words, the values of x_1 and x_2 being assumed at pleasure, x_3, x_4, and x_5 can be so determined as to satisfy all ten equations.

I. Solution by Arnold Dresden, University of Wisconsin.

Denote by E_i the function $x_i + x_{i+2}x_{i+3} - 1$ and by $\bar{E}_i$ the function $x_ix_{i+1}x_{i+2} - x_i - x_{i+2} + 1$. Then the proposed equations may be written in the form $E_i = 0$, $\bar{E}_i = 0$, $(i = 1, \cdots, 5)$, the subscripts being reduced modulo 5.

We have the following identities:

$$(1)\qquad \begin{cases} x_{i+1}E_i - E_{i+3} \equiv \bar{E}_{i+1} \\ x_{i-1}E_i - E_{i+2} \equiv \bar{E}_{i+2}, \end{cases}$$

so that the equations $\bar{E}_i = 0$ are dependent upon the equations $E_i = 0$.

But from the relation (1), we derive, moreover, the following

$$(5)\qquad E_{i-1} + x_{i-1}E_i \equiv x_{i+2}E_{i+1} + E_{i+2}.$$

Hence from any three of the equations, $E_i = 0$, the remaining two must follow.

II. Solution by Horace L. Olson, Heidelberg University, Tiffin, Ohio.

The ten equations mentioned are

$$\begin{array}{llll} (1) & x_1 = 1 - x_3x_4 & (6) & x_5x_1x_2 = x_5 + x_2 - 1 \\ (2) & x_2 = 1 - x_4x_5 & (7) & x_1x_2x_3 = x_1 + x_3 - 1 \\ (3) & x_3 = 1 - x_5x_1 & (8) & x_2x_3x_4 = x_2 + x_4 - 1 \\ (4) & x_4 = 1 - x_1x_2 & (9) & x_3x_4x_5 = x_3 + x_5 - 1 \\ (5) & x_5 = 1 - x_2x_3 & (10) & x_4x_5x_1 = x_4 + x_1 - 1. \end{array}$$

Let us assume, first, that $x_1x_2 \neq 1$. Then from equations (4), (2), and (3), respectively, we find that,

$$x_4 = 1 - x_1x_2, \qquad x_5 = (1 - x_2)/(1 - x_1x_2), \qquad x_3 = (1 - x_1)/(1 - x_1x_2).$$